61A Lecture 20

Announcements

Sets

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...            open('shakespeare.txt').read())
>>> W = set(text)
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One more built-in Python container type

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>>> s.union({1, 5})
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(Demo)

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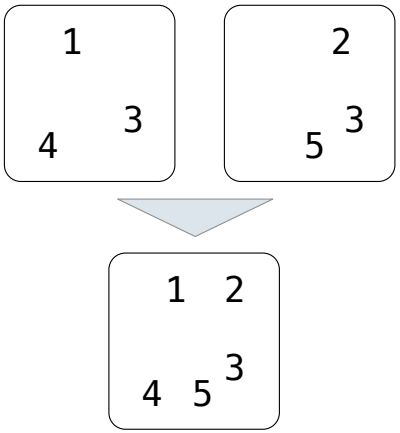
Union



Implementing Sets

What we should be able to do with a set:

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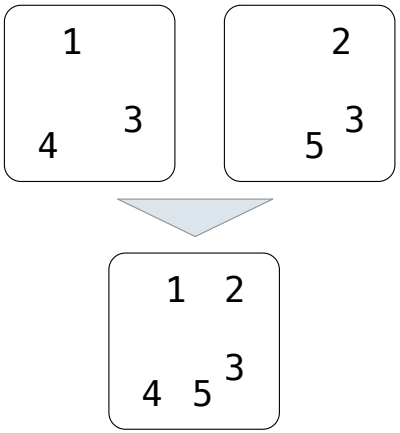
- **Intersection:** Return a set with any elements in set1 and set2

Union



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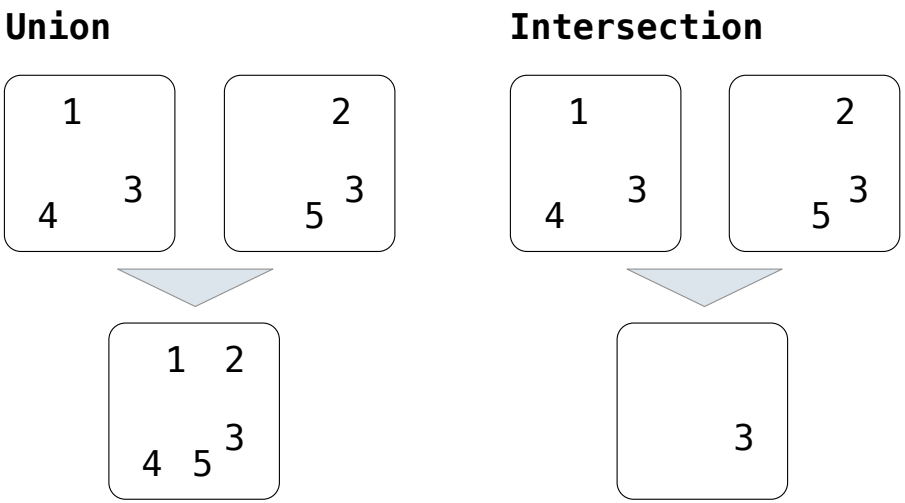

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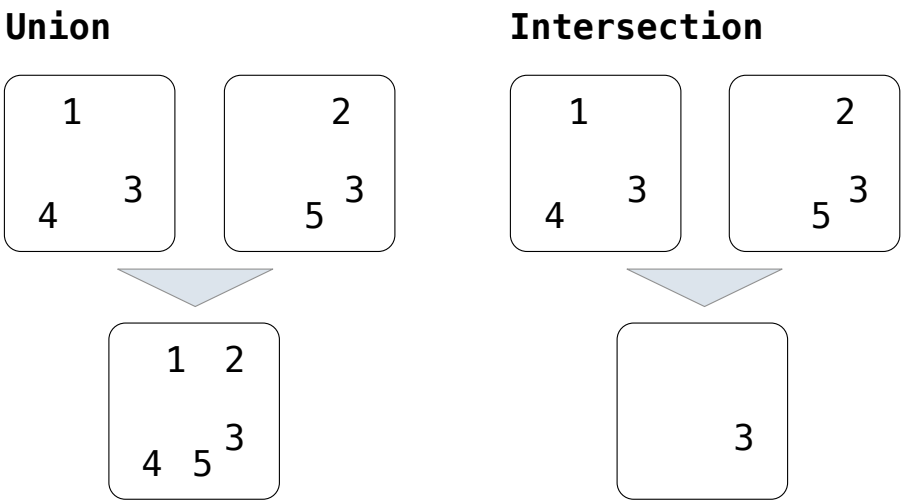

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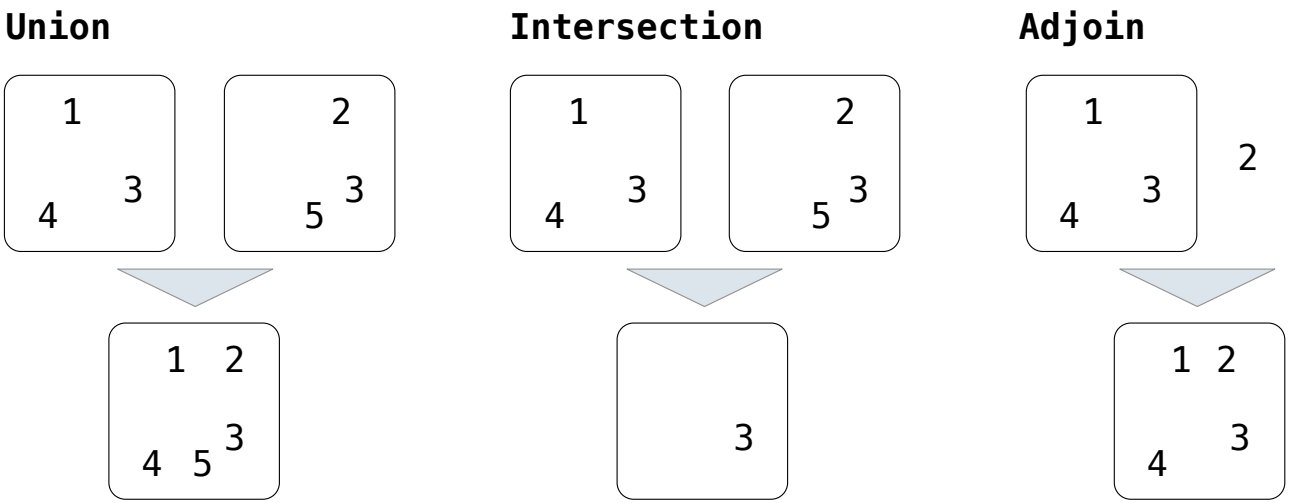

Sets as Linked Lists

Sets as Unordered Sequences

Proposal 1: A set is represented by a linked list that contains no duplicate items.

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In the worst case: v does not appear in s

or

In the average case: appears in a uniformly distributed random location

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def adjoin(s, v):
    if contains(s, v):
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def adjoin(s, v):
    if contains(s, v):
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def intersect(set1, set2):
    in_set2 = lambda v: contains(set2, v)
    return filter_link(in_set2, set1)
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def union(set1, set2):
    not_in_set2 = lambda v: not contains(set2, v)
    set1_not_set2 = filter_link(not_in_set2, set1)
    return extend_link(set1_not_set2, set2)
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Return elements x for which in_set2(x) returns a true value

Return a linked list containing all elements in set1_not_set2 followed by all elements in set2

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Parts of the program that...	Assume that sets are...	Using...
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Parts of the program that...	Assume that sets are...	Using...
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Use sets to contain values	Unordered collections	`empty`, `contains`, `adjoin`, `intersect`, `union`
Implement set operations	Ordered linked lists	`first`, `rest`, `<`, `>`, `==`

Different parts of a program may make different assumptions about data

Searching an Ordered List

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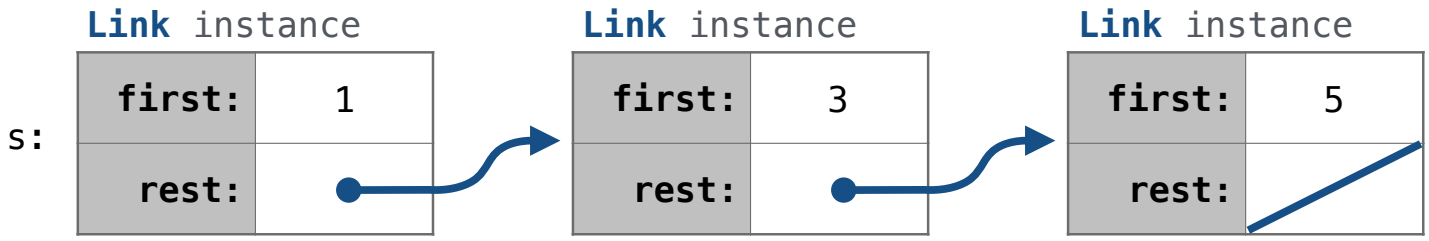

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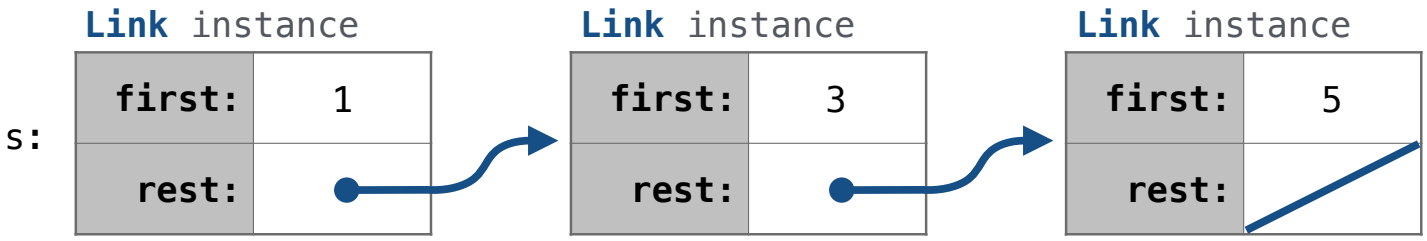
Operation	Time order of growth



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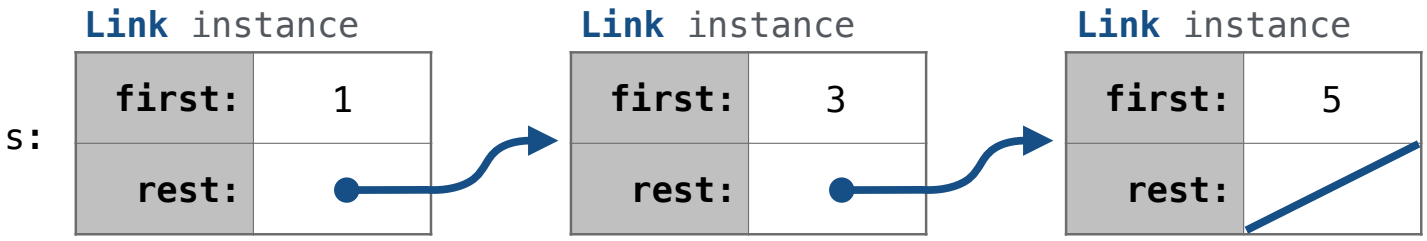

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>>> contains(s, 1)
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Operation	Time order of growth
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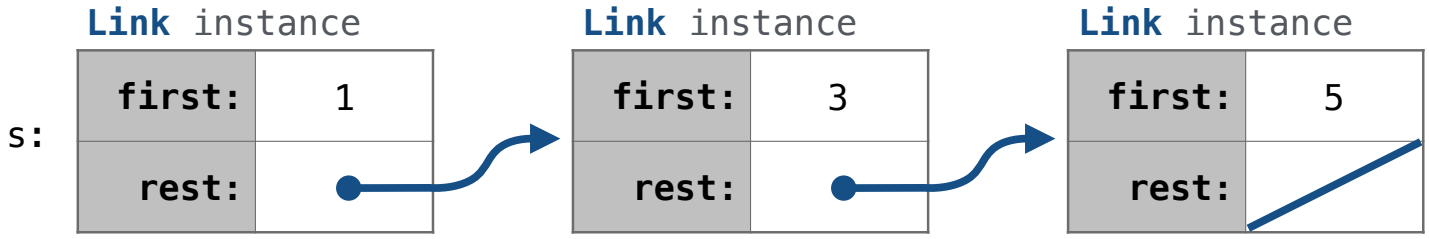

Searching an Ordered List

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>>> s = Link(1, Link(3, Link(5)))
>>> contains(s, 1)
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Operation	Time order of growth
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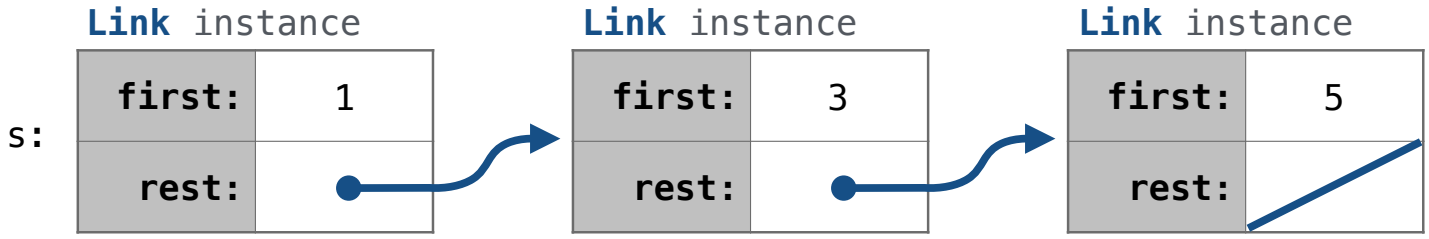

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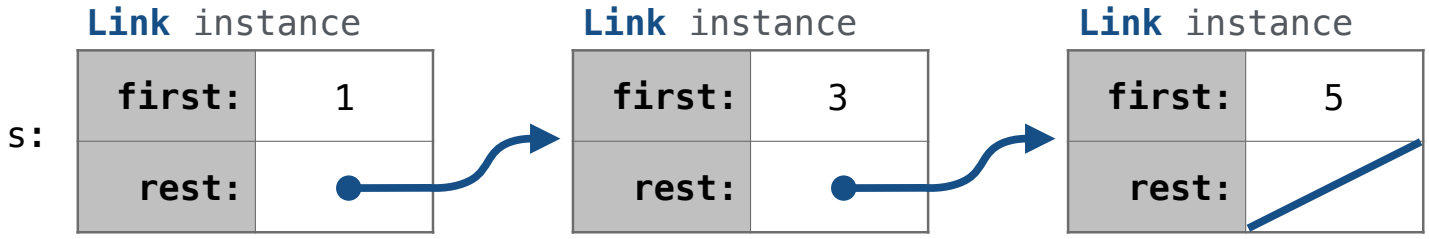
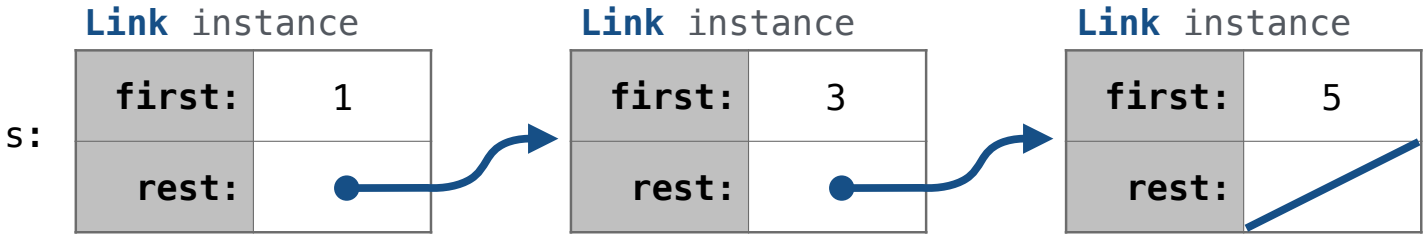
Operation	Time order of growth
contains	



Searching an Ordered List

```
>>> s = Link(1, Link(3, Link(5)))
>>> contains(s, 1)
True
>>> contains(s, 2)
False
```

Operation	Time order of growth
contains	



Searching an Ordered List

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>>> s = Link(1, Link(3, Link(5)))
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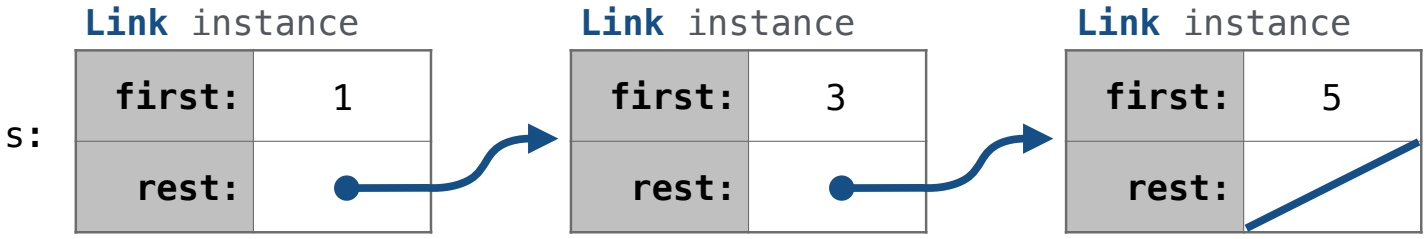
```

Operation	Time order of growth
`contains`	$\Theta(n)$



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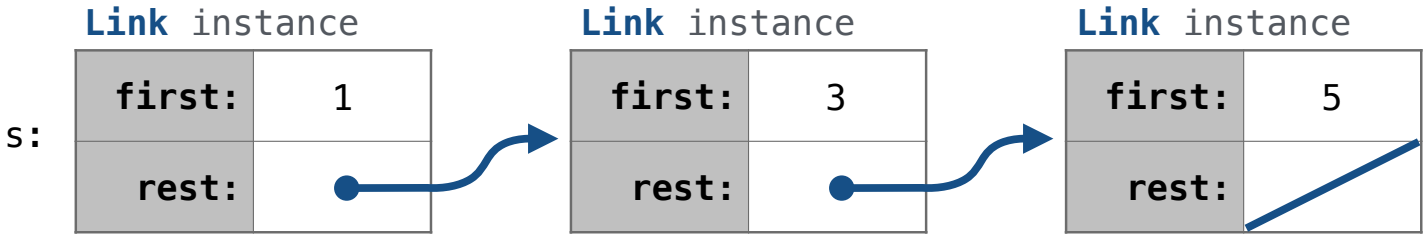
```

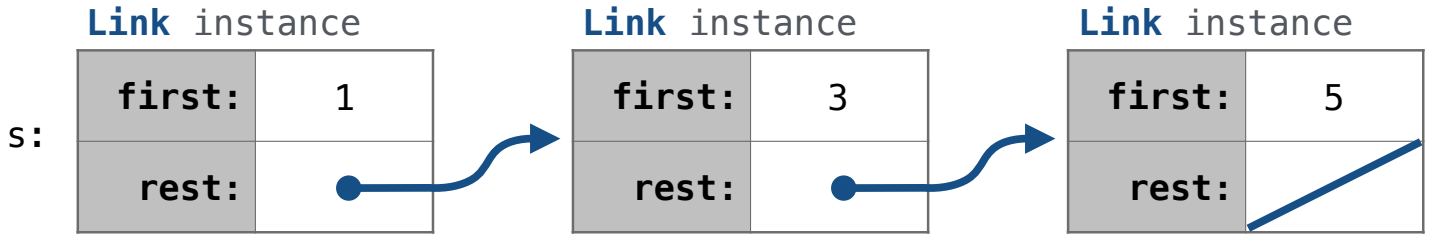
Operation	Time order of growth
`contains`	$\Theta(n)$
`adjoin`	



Searching an Ordered List

```
>>> s = Link(1, Link(3, Link(5)))
>>> contains(s, 1)
True
>>> contains(s, 2)
False
>>> t = adjoin(s, 2)
```

Operation	Time order of growth
`contains`	$\Theta(n)$
`adjoin`	



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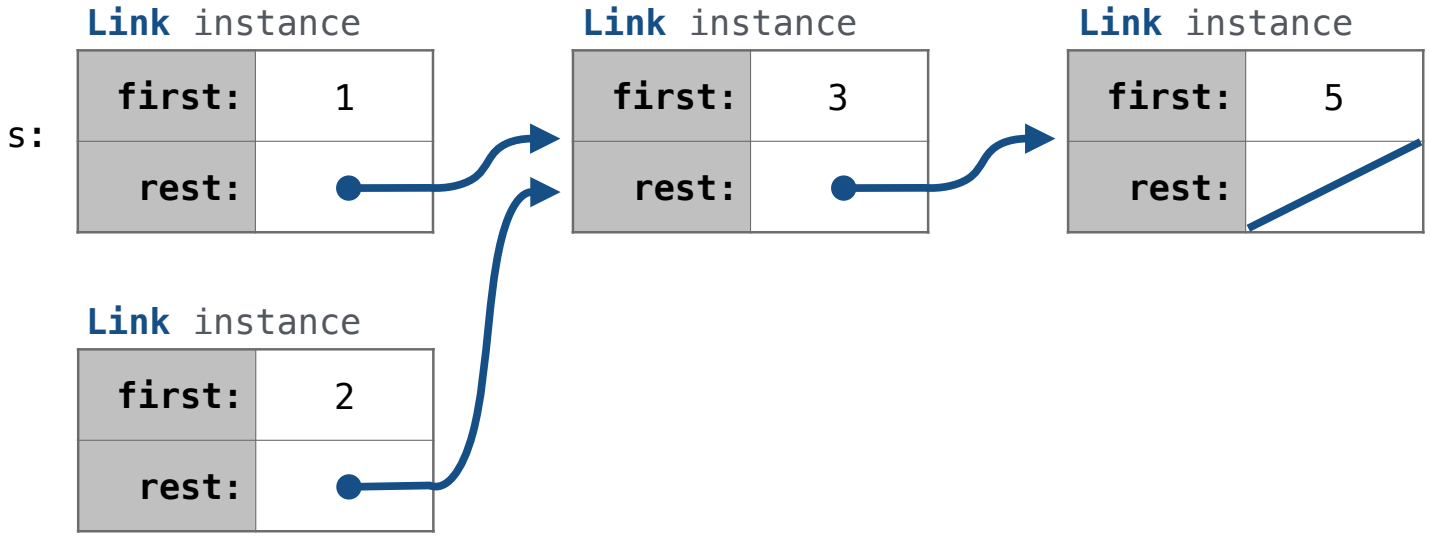

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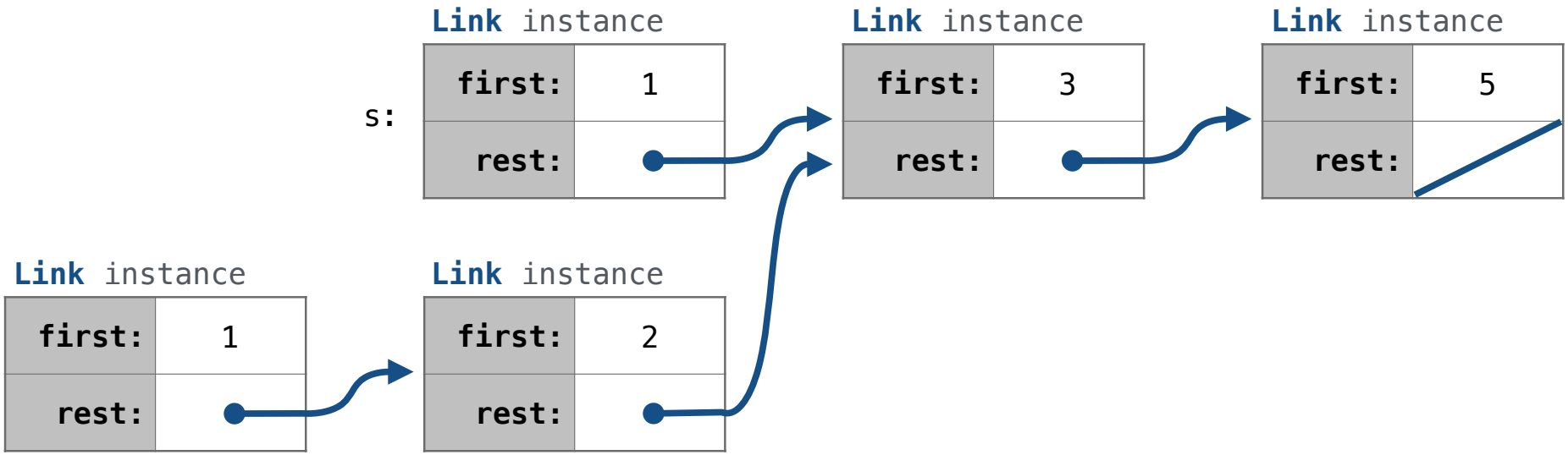

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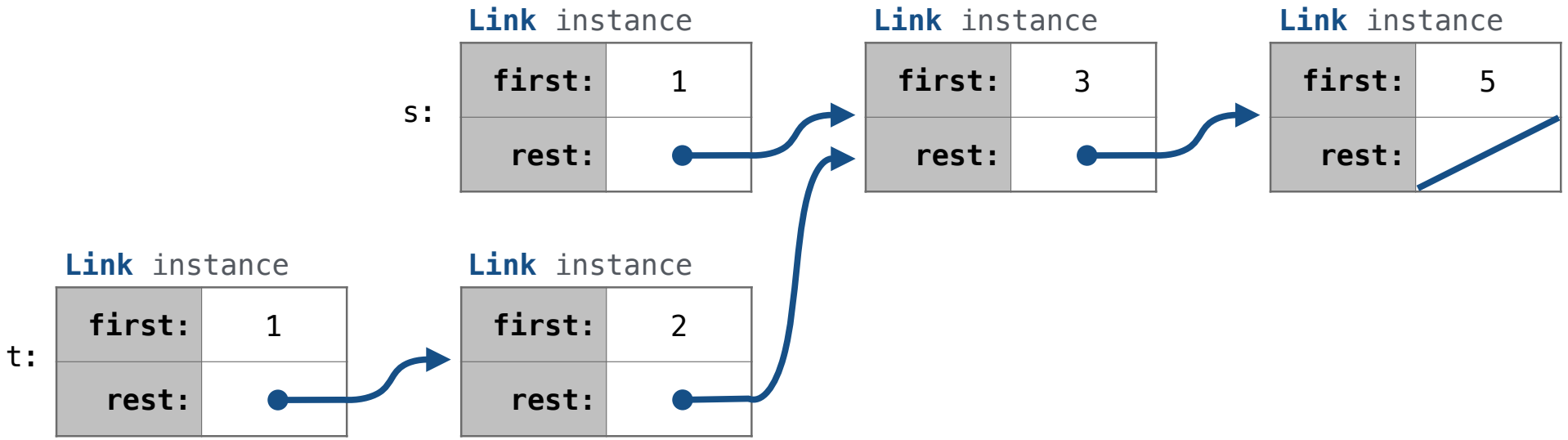

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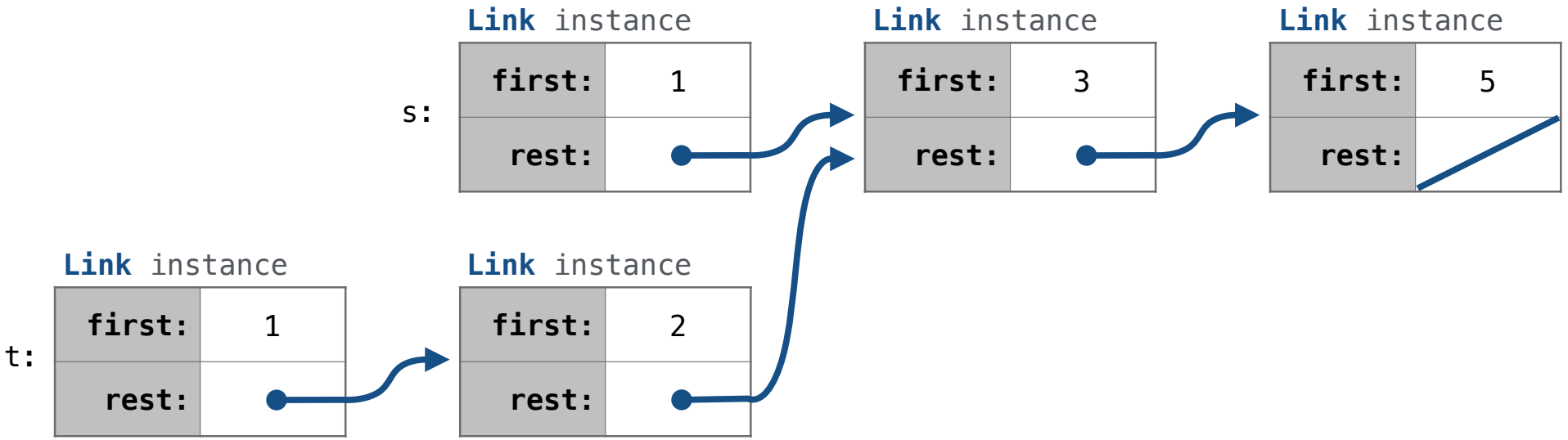

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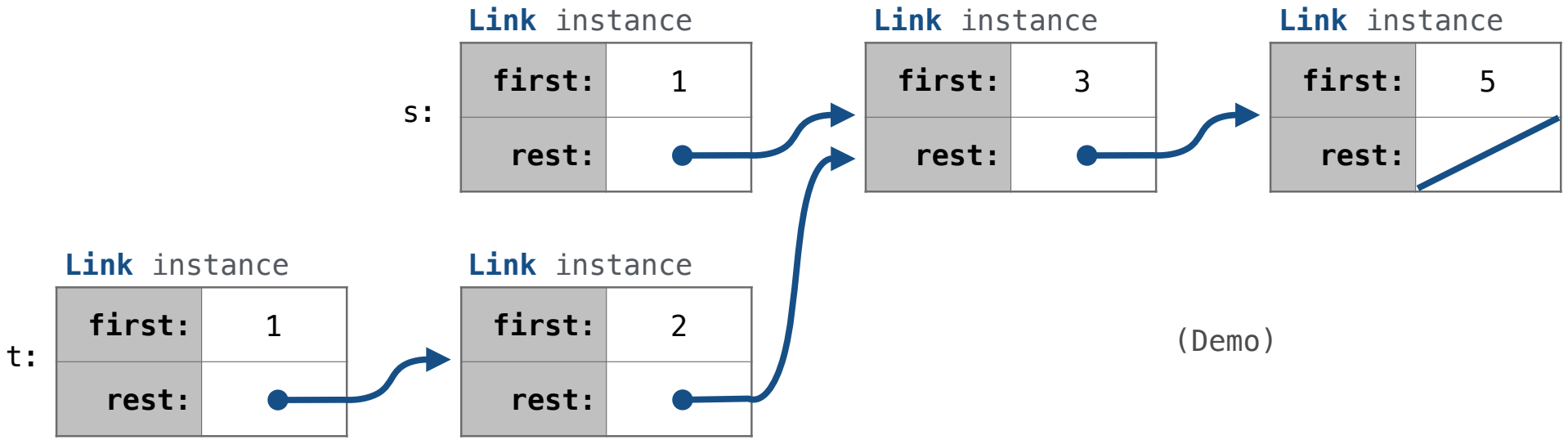

(Demo)

Set Operations

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

Let n be max of set1, set2 size

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

```
def intersect(set1, set2):
```

Let n be max of set1, set2 size

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

```
def intersect(set1, set2):
    if empty(set1) or empty(set2):
        return Link.empty
```

Let n be max of set1, set2 size

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

```
def intersect(set1, set2):
    if empty(set1) or empty(set2):
        return Link.empty
    else:
```

Let *n* be max of set1, set2 size

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

```
def intersect(set1, set2):
    if empty(set1) or empty(set2):
        return Link.empty
    else:
        e1, e2 = set1.first, set2.first
```

Let n be max of set1, set2 size

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

```
def intersect(set1, set2):
    if empty(set1) or empty(set2):
        return Link.empty
    else:
        e1, e2 = set1.first, set2.first
        if e1 == e2:
            return Link(e1, intersect(set1.rest, set2.rest))
```

Let n be max of set1, set2 size

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

```
def intersect(set1, set2):
    if empty(set1) or empty(set2):
        return Link.empty
    else:
        e1, e2 = set1.first, set2.first
        if e1 == e2:
            return Link(e1, intersect(set1.rest, set2.rest))
        elif e1 < e2:
            return intersect(set1.rest, set2)
```

Let n be max of set1, set2 size

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

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def intersect(set1, set2):
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        if e1 == e2:
            return Link(e1, intersect(set1.rest, set2.rest))
        elif e1 < e2:
            return intersect(set1.rest, set2)
        elif e2 < e1:
            return intersect(set1, set2.rest)
```

Let n be max of set1, set2 size

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

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def intersect(set1, set2):
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    else:
        e1, e2 = set1.first, set2.first
        if e1 == e2:
            return Link(e1, intersect(set1.rest, set2.rest))
        elif e1 < e2:
            return intersect(set1.rest, set2)
        elif e2 < e1:
            return intersect(set1, set2.rest)
```

Let n be max of set1, set2 size

Order of growth?

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

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        if e1 == e2:
            return Link(e1, intersect(set1.rest, set2.rest))
        elif e1 < e2:
            return intersect(set1.rest, set2)
        elif e2 < e1:
            return intersect(set1, set2.rest)
```

Let n be max of set1, set2 size

Order of growth? $\Theta(n)$

Intersecting Ordered Linked Lists

Proposal 2: A set is represented by a linked list with unique elements that is ***ordered from least to greatest***

```
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    else:
        e1, e2 = set1.first, set2.first
        if e1 == e2:
            return Link(e1, intersect(set1.rest, set2.rest))
        elif e1 < e2:
            return intersect(set1.rest, set2)
        elif e2 < e1:
            return intersect(set1, set2.rest)
```

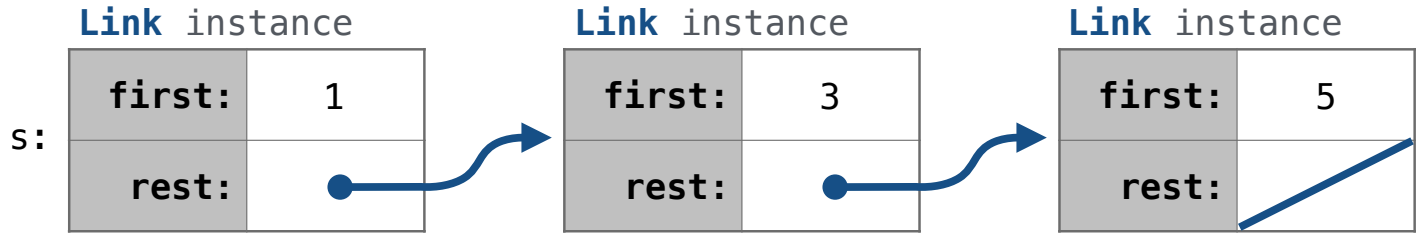
Let n be max of set1, set2 size

Order of growth? $\Theta(n)$

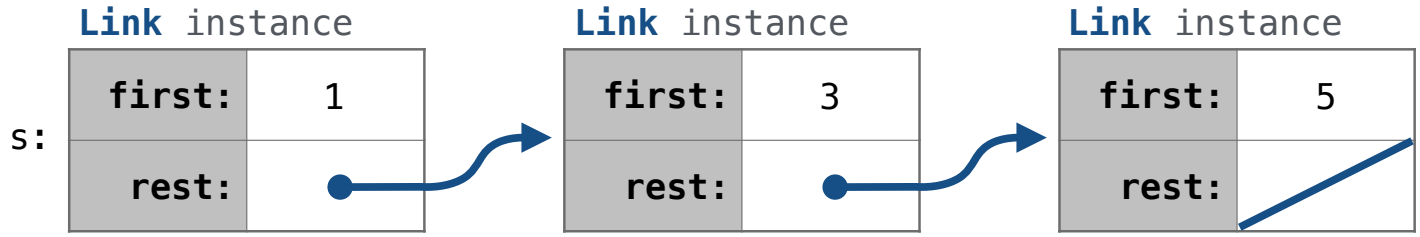
(Demo)

Set Mutation

Adding to an Ordered List

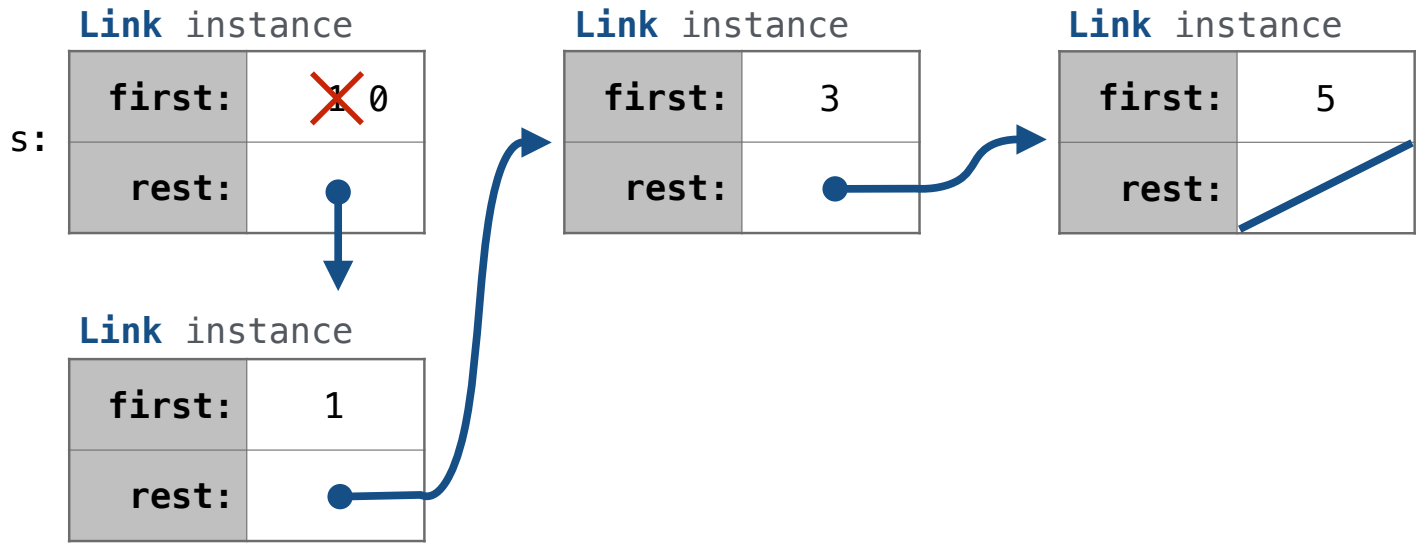


Adding to an Ordered List

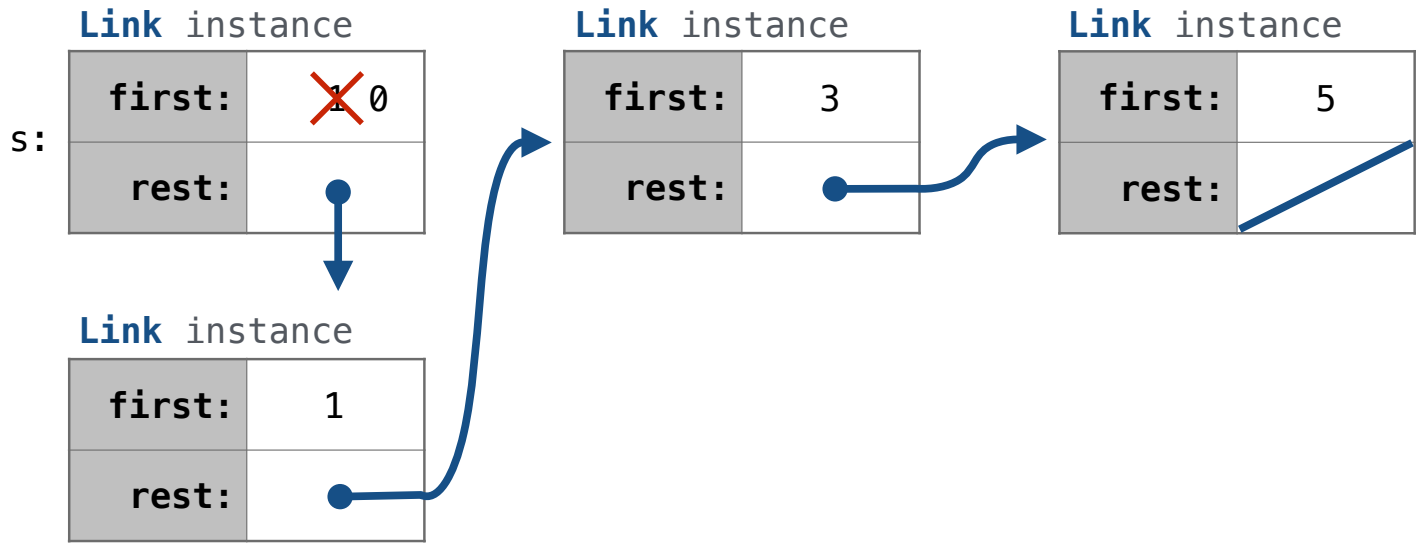



```
add(s, 0)   Try to return the same object as input
```

Adding to an Ordered List

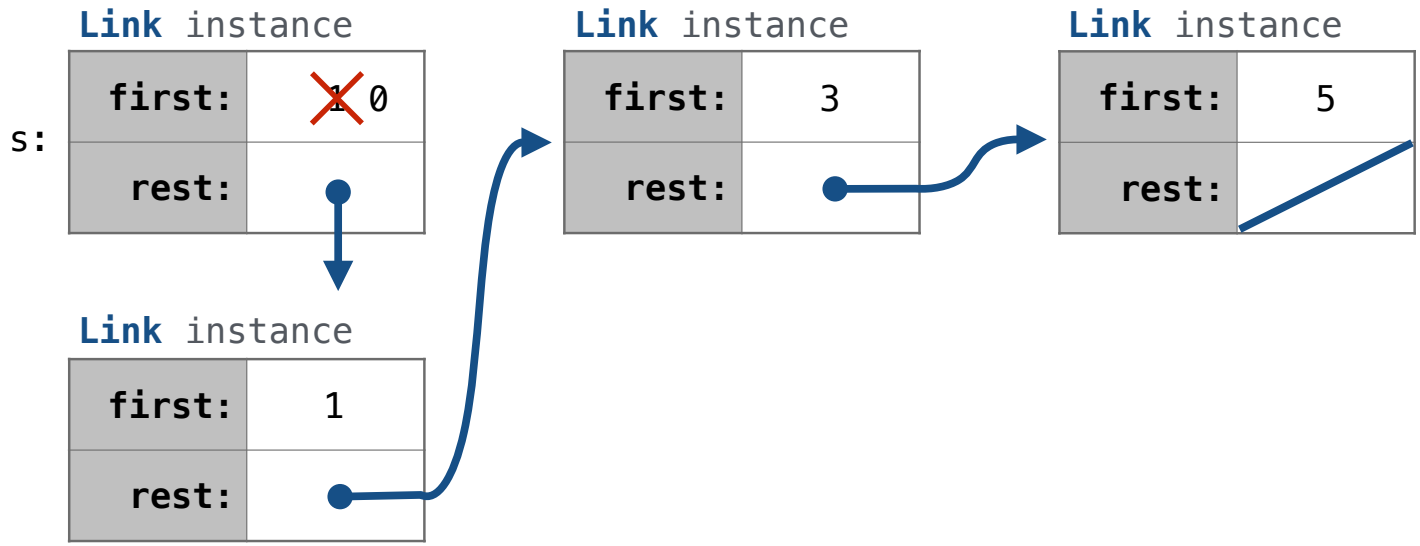


Adding to an Ordered List



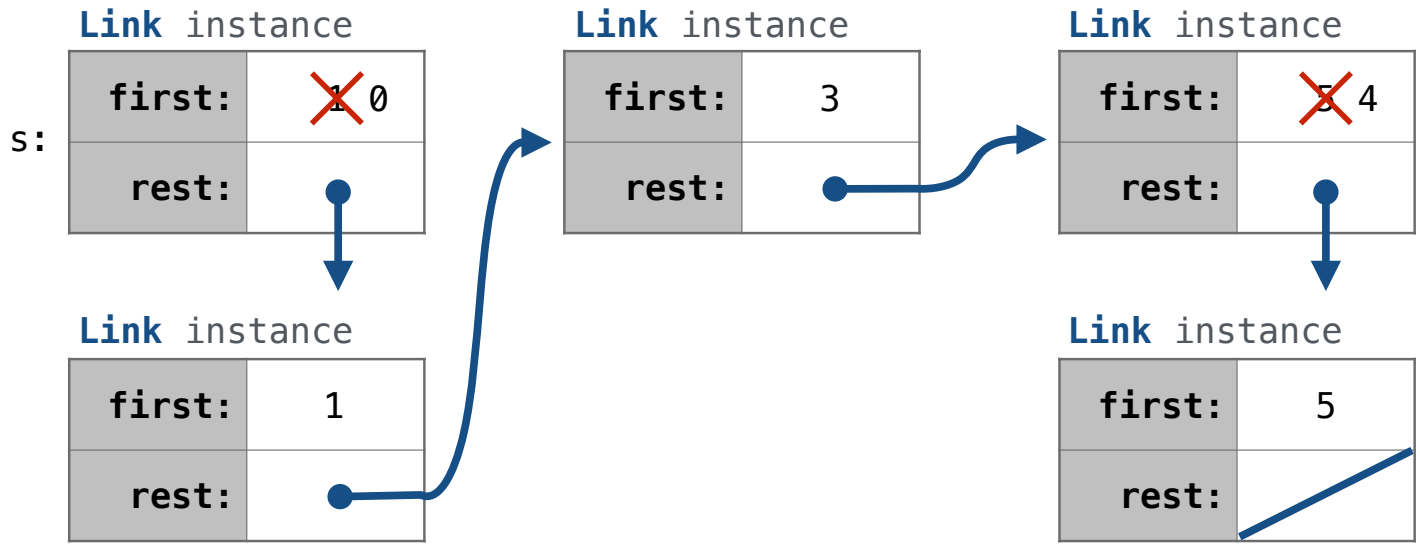

```
add(s, 3)
```

Adding to an Ordered List

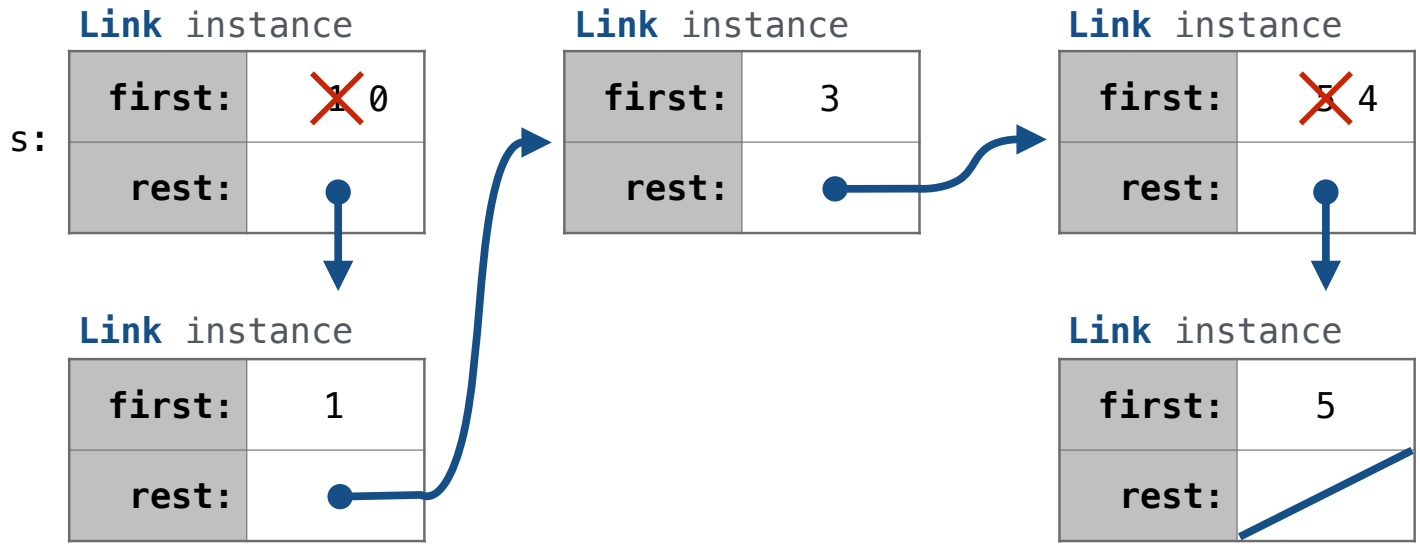



```
add(s, 3)
add(s, 4)
```

Adding to an Ordered List

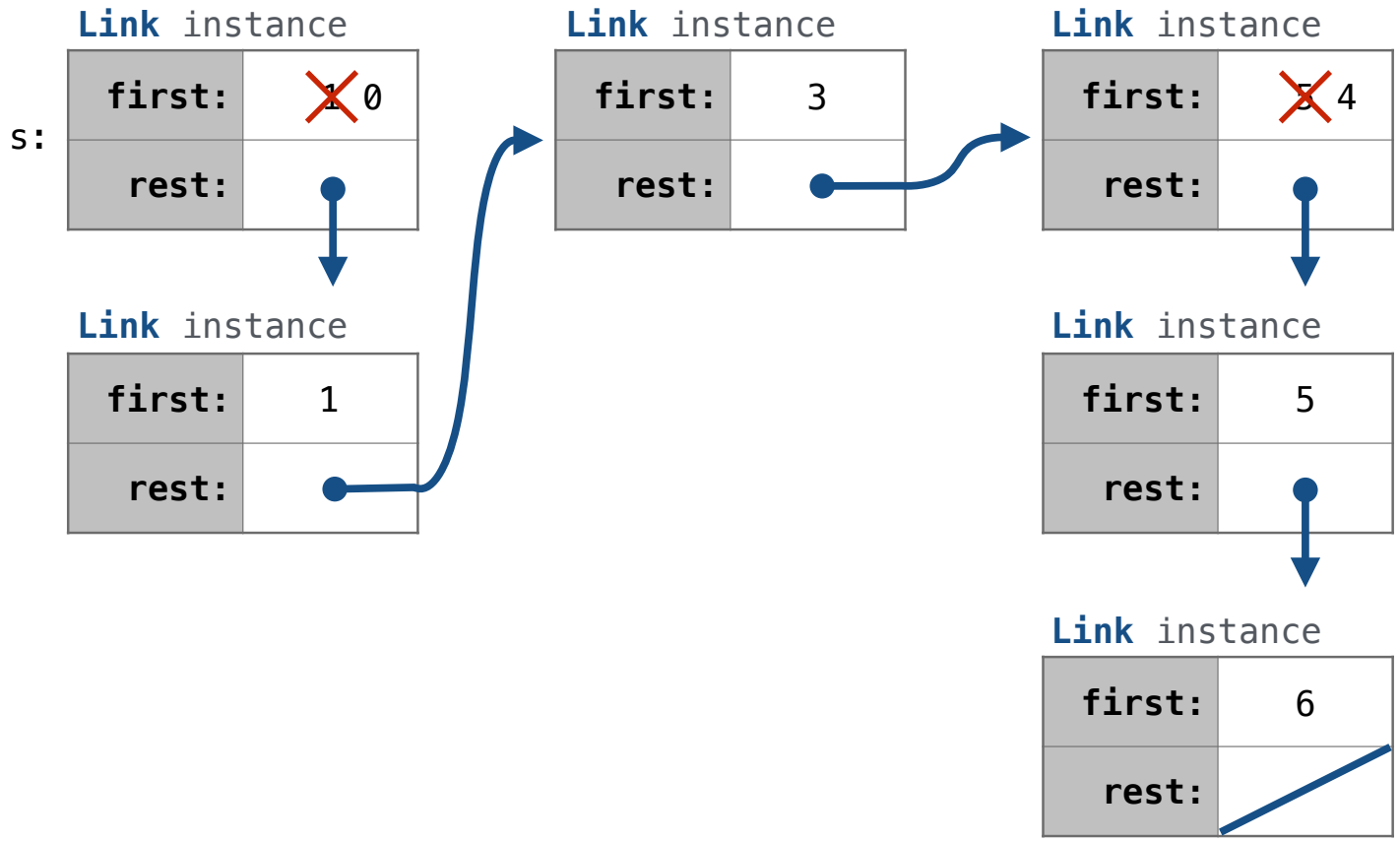


Adding to an Ordered List

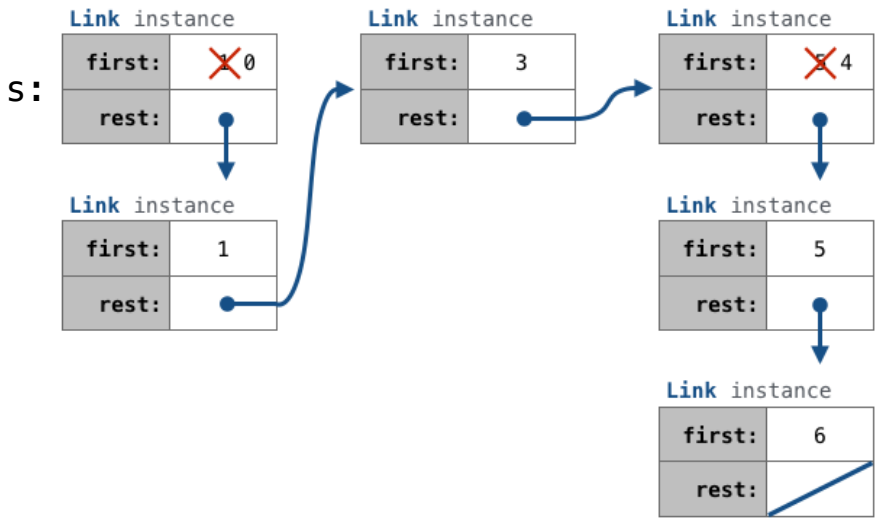



```
add(s, 6)
```

Adding to an Ordered List

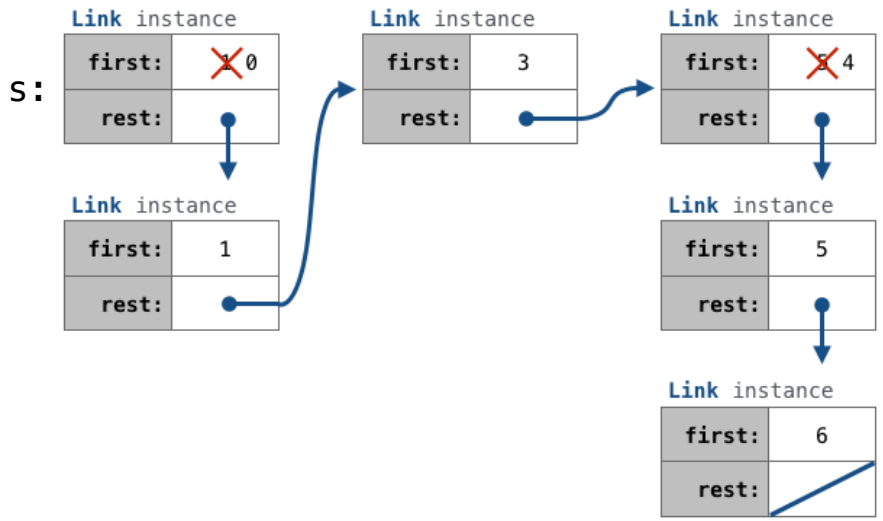


Adding to a Set Represented as an Ordered List



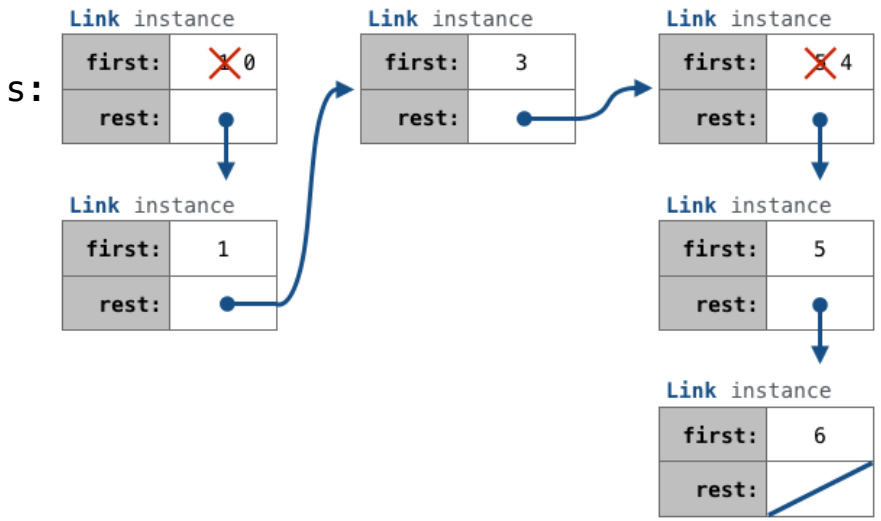
Adding to a Set Represented as an Ordered List

```
def add(s, v):
```



Adding to a Set Represented as an Ordered List

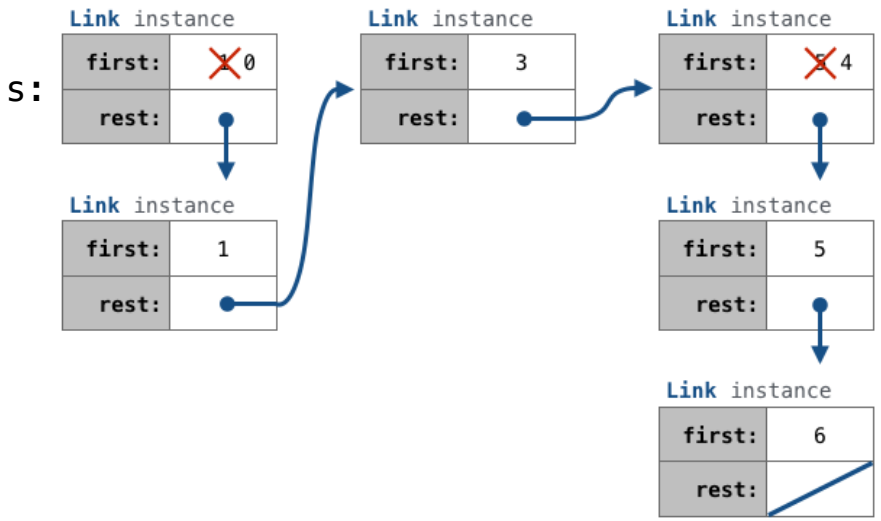
```
def add(s, v):
    """Add v to a set s, returning modified s."""
```



Adding to a Set Represented as an Ordered List

```
def add(s, v):
    """Add v to a set s, returning modified s."""

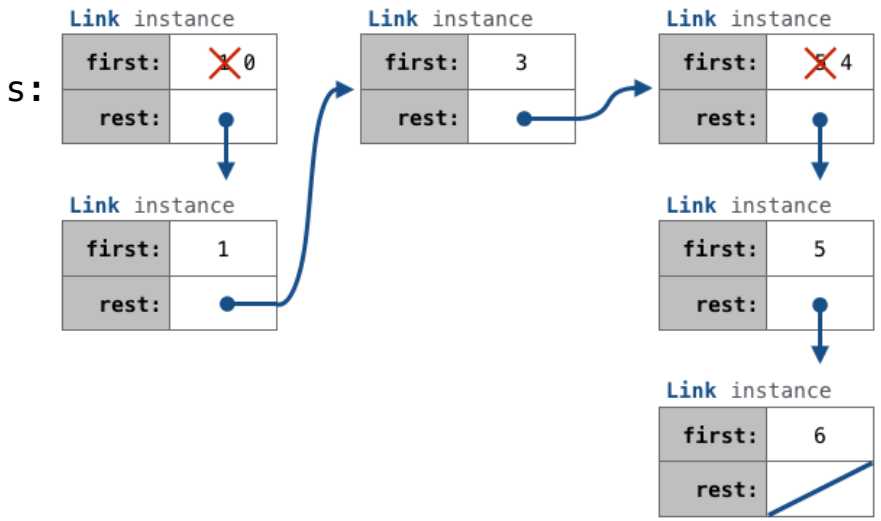
    >>> s = Link(1, Link(3, Link(5)))
```



Adding to a Set Represented as an Ordered List

```
def add(s, v):
    """Add v to a set s, returning modified s."""

    >>> s = Link(1, Link(3, Link(5)))
    >>> add(s, 0)
    Link(0, Link(1, Link(3, Link(5))))
```



Adding to a Set Represented as an Ordered List

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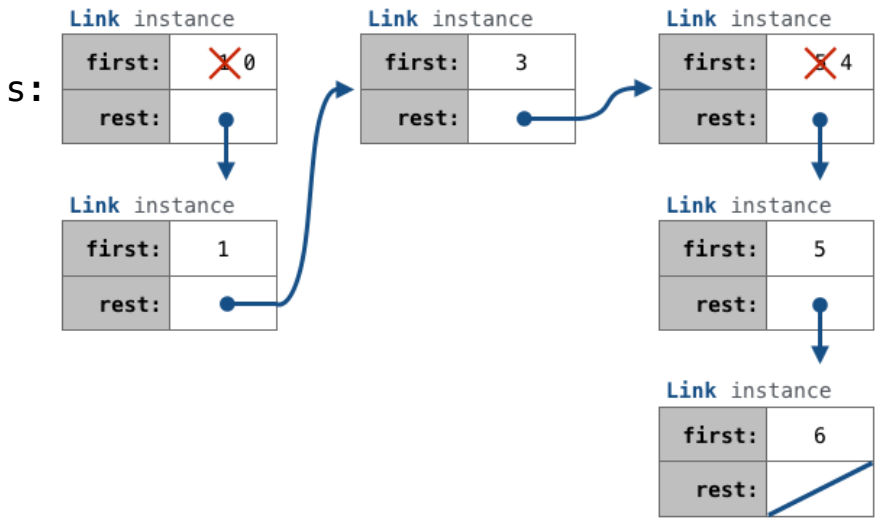
    >>> s = Link(1, Link(3, Link(5)))
    >>> add(s, 0)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 3)
    Link(0, Link(1, Link(3, Link(5))))
```



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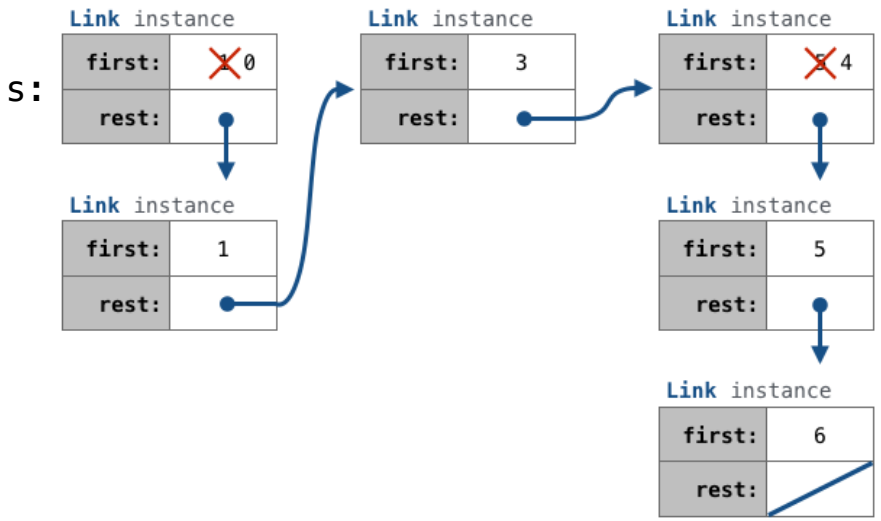
    >>> add(s, 3)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
```



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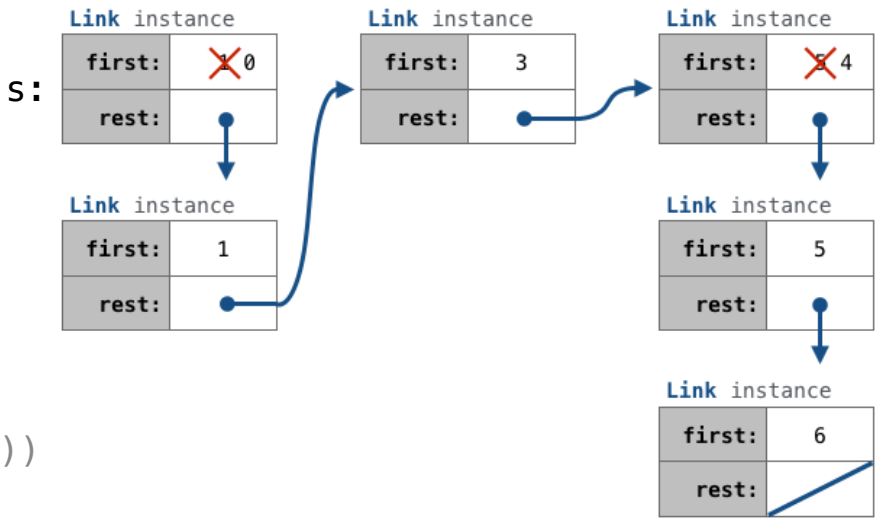
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))
    """
```



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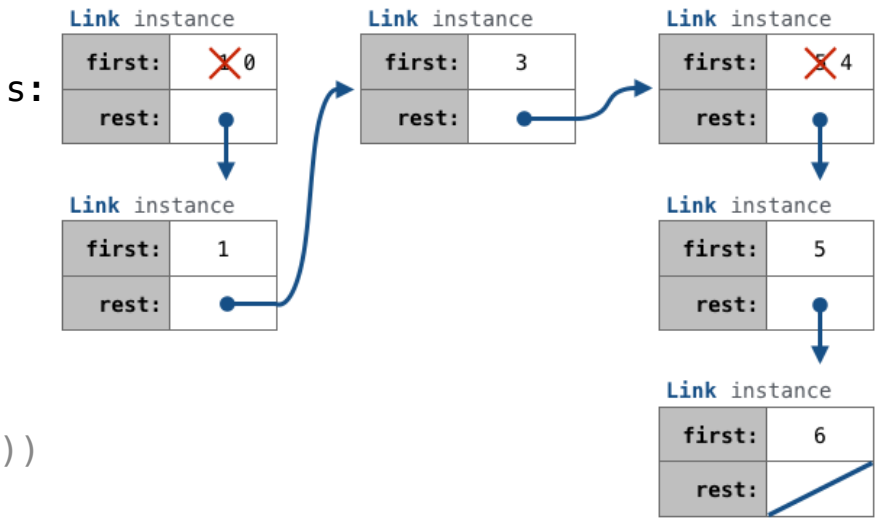
    >>> add(s, 3)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))))
    """
    if empty(s): return Link(v)
```



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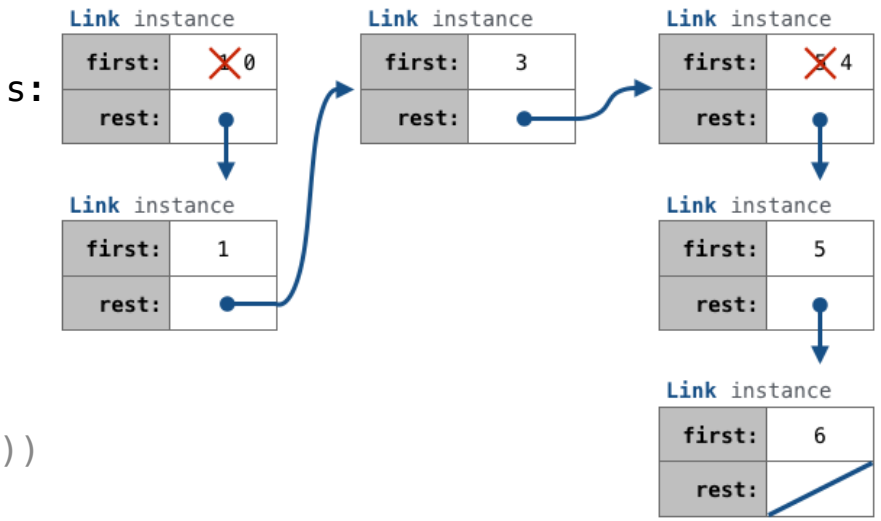
    >>> add(s, 3)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))))
    """

    if s.first > v:
        s.first, s.rest = _____________________________ , _____________________________
```



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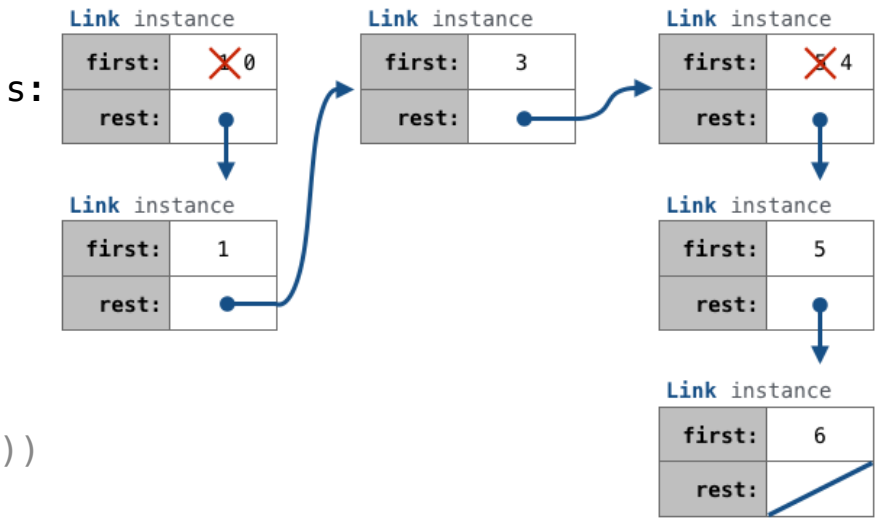
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))))
    """

    if s.first > v:
        s.first, s.rest = _____________________________ , _____________________________
    elif s.first < v and empty(s.rest):
        s.rest = ____________________________________________________________________
```



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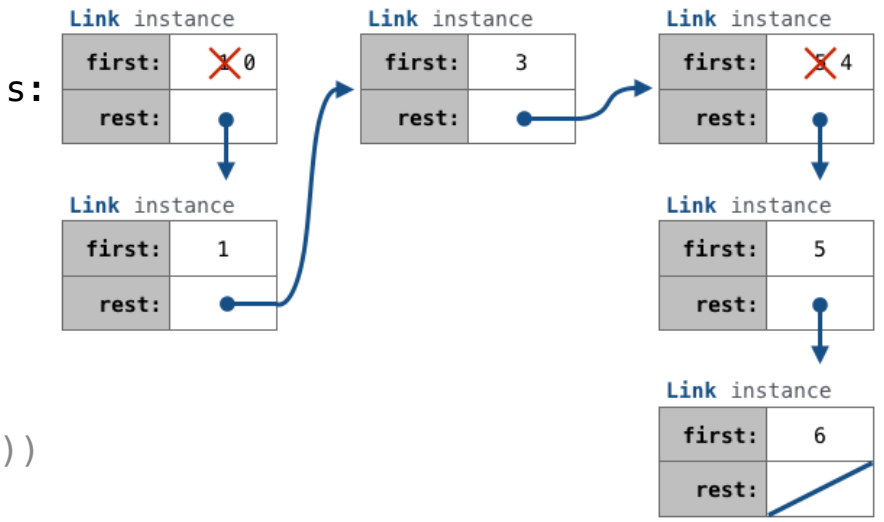
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))))
    """

    if s.first > v:
        s.first, s.rest = _____________________________ , _____________________________
    elif s.first < v and empty(s.rest):
        s.rest = ____________________________________________________________________
    elif s.first < v:
        ____________________________________________________________________________
    return s
```



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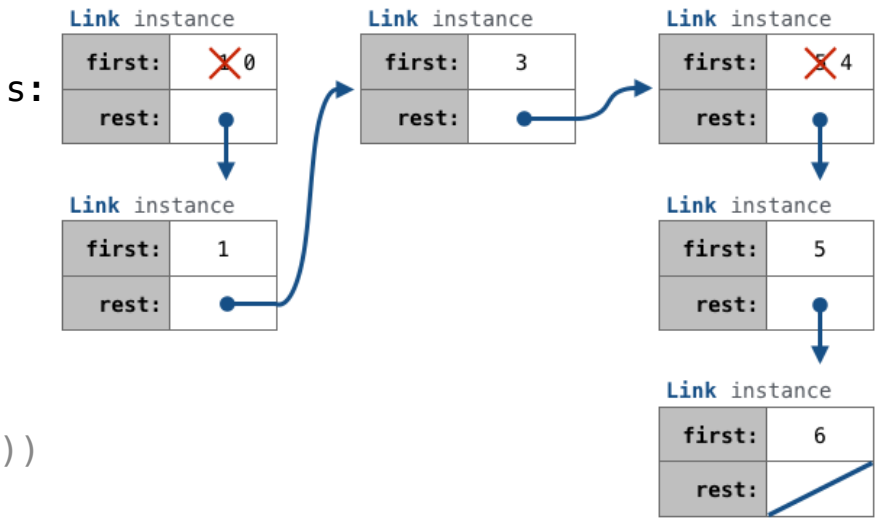
    >>> add(s, 3)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
    >>> add(s, 6)
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))))
    """

    if s.first > v:
        s.first, s.rest = _____v_____ , _____________________________
    elif s.first < v and empty(s.rest):
        s.rest = ____________________________________________________________
    elif s.first < v:

        ____________________________________________________________________
    return s
```



Adding to a Set Represented as an Ordered List

```
def add(s, v):
    """Add v to a set s, returning modified s."""

    >>> s = Link(1, Link(3, Link(5)))
    >>> add(s, 0)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 3)
    Link(0, Link(1, Link(3, Link(5))))
    >>> add(s, 4)
    Link(0, Link(1, Link(3, Link(4, Link(5)))))
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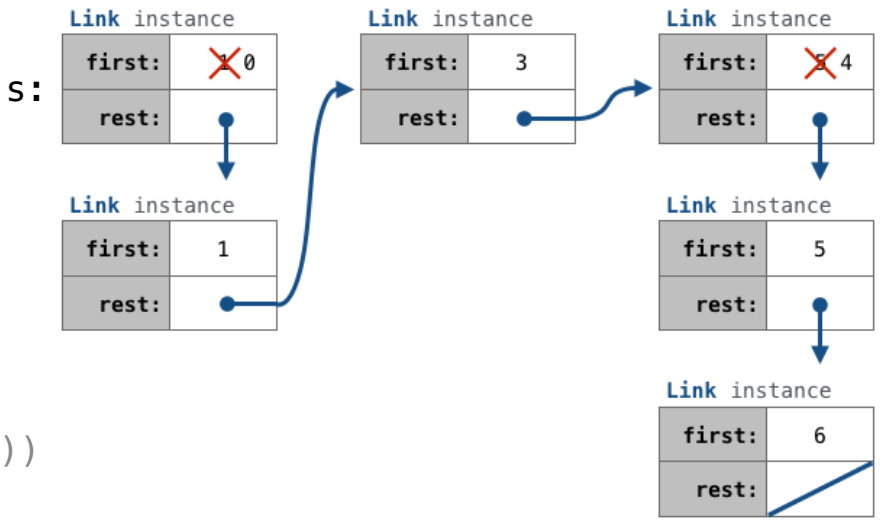
    Link(0, Link(1, Link(3, Link(4, Link(5, Link(6))))
    """

    if s.first > v:
        s.first, s.rest = _____v_____ , _____Link(s.first, s.rest)_____
    elif s.first < v and empty(s.rest):
        s.rest = _____________________________________________________________
    elif s.first < v:
        _____________________________________________________________________
    return s
```



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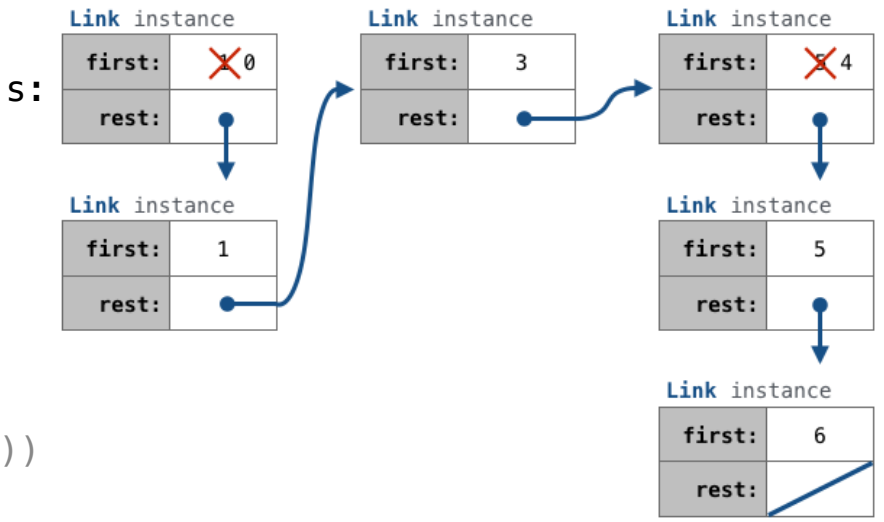
    """

    if s.first > v:
        s.first, s.rest = ____v____ , ____Link(s.first, s.rest)____
    elif s.first < v and empty(s.rest):
        s.rest = ____Link(v, s.rest)____
    elif s.first < v:
        ____
    return s
```



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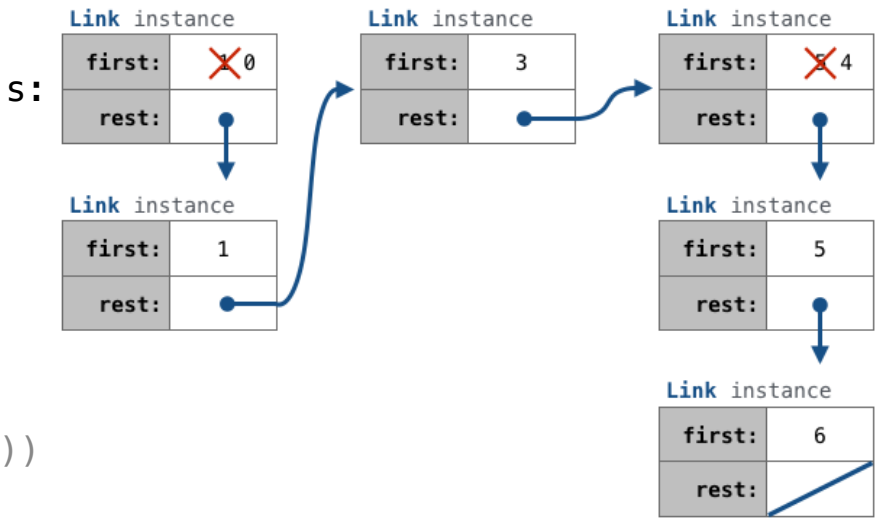


```
    if s.first > v:
        s.first, s.rest = ____v____ , ____Link(s.first, s.rest)____
    elif s.first < v and empty(s.rest):
        s.rest = ____Link(v, s.rest)____
    elif s.first < v:
        ____add(s.rest, v)____
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    elif s.first < v and empty(s.rest):
        s.rest = ____Link(v, s.rest)____
    elif s.first < v:
        ____add(s.rest, v)____
    return s
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